浅水(波)方程式 から 渦位保存則 を導く

①, ②, ④ ⑥

$\frac{du}{dt}$

$\times -\frac{\partial}{\partial y}$ $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \eta}{\partial x}$ ——①

$\times \frac{\partial}{\partial x}$ $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial \eta}{\partial y}$ ——②] 運動方程式

ζ ζ ζ

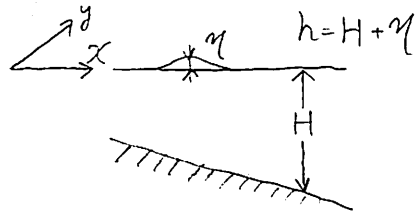
$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + f\left(\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right)$

$-\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right) - \left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial u}{\partial y}\right) - \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right) = 0$

$\frac{d}{dt}\zeta + \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$

$\frac{df}{dt}=0$ $\frac{d}{dt}(\zeta + f) + (f+\zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$ —— ③

$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0$ —— ④ 連続の式



$\frac{\partial \eta}{\partial t} + u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$

$\frac{\partial H}{\partial t}=0$ $\frac{\partial(H+\eta)}{\partial t} + u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$

$H \gg \eta$, $h = H+\eta$ $\frac{dh}{dt} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$ —— ⑤

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③と⑤から $\frac{d(f+\zeta)}{dt} - (f+\zeta)\frac{1}{h}\frac{dh}{dt} = 0$

$\frac{d}{dt}\left(\frac{1}{h}\right) = \frac{dh}{dt}\cdot\left(-\frac{1}{h^2}\right)$ $\frac{d(f+\zeta)}{dt} + (f+\zeta)h\frac{d}{dt}\left(\frac{1}{h}\right) = 0$

$\frac{1}{h}\frac{d(f+\zeta)}{dt} + (f+\zeta)\frac{d}{dt}\left(\frac{1}{h}\right) = 0$

$\frac{d}{dt}\left[\frac{f+\zeta}{h}\right] = 0$ —— ⑥ 渦位保存則